

GLOBAL
EDITION



Calculus

Early Transcendentals

SECOND EDITION

William Briggs • Lyle Cochran • Bernard Gillett

Calculus

EARLY TRANSCENDENTALS

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Calculus

EARLY TRANSCENDENTALS

GLOBAL EDITION

Second Edition

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Preface

The second edition of *Calculus: Early Transcendentals* supports a three-semester or four-quarter calculus sequence typically taken by students studying mathematics, engineering, the natural sciences, or economics. The second edition has the same goals as the first edition:

- to motivate the essential ideas of calculus with a lively narrative, demonstrating the utility of calculus with applications in diverse fields;
- to introduce new topics through concrete examples, applications, and analogies, appealing to students' intuition and geometric instincts to make calculus natural and believable; and
- once this intuitive foundation is established, to present generalizations and abstractions and to treat theoretical matters in a rigorous way.

The second edition both builds on the success and addresses the inevitable deficiencies of the first edition. We have listened to and learned from the instructors who used the first edition. They have given us wise guidance about how to make the second edition an even more effective learning tool for students and a more powerful resource for instructors. Users of the book continue to tell us that it mirrors the course they teach—and more importantly, that students actually read it! Of course, the second edition also benefits from our own experiences using the book, as well as our experiences teaching mathematics at diverse institutions over the past 30 years.

We are grateful to users of the first edition—for their courage in adopting a first edition book, for their enthusiastic response to the book, and for their invaluable advice and feedback. They deserve much of the credit for the improvements that we have made in the second edition.

New in the Second Edition

Narrative

The second edition of this book has undergone a thorough cover-to-cover polishing of the narrative, making the presentation of material even more concise and lucid. Occasionally, we discovered new ways to present material to make the exposition clearer for students and more efficient for instructors.

Figures

The figures were thoroughly reviewed and revised when necessary. The figures enrich the overall spirit of the book and tell as much of the calculus story as the words do.

Exercises

The comprehensive 7656 exercises in the first edition were thoroughly reviewed and refined. Then 19% more basic skills and mid-level exercises were added. The exercises at the end of each section are still efficiently organized in the following categories.

- *Review Questions* begin each exercise set and check students' conceptual understanding of the essential ideas from the section.
- *Basic Skills* exercises are confidence-building problems that provide a solid foundation for the more challenging exercises to follow. Each example in the narrative is linked directly to a block of *Basic Skills* exercises via *Related Exercises* references at the end of the example solution.
- *Further Explorations* exercises expand on the *Basic Skills* exercises by challenging students to think creatively and to generalize newly acquired skills.
- *Applications* exercises connect skills developed in previous exercises to applications and modeling problems that demonstrate the power and utility of calculus.
- *Additional Exercises* are generally the most difficult and challenging problems; they include proofs of results cited in the narrative.

Each chapter concludes with a comprehensive set of *Review Exercises*.

Answers

The answers in the back of the book have been reviewed and thoroughly checked for accuracy. The reliability that we achieved in the first edition has been maintained—if not improved.

New Topics

We have added new material on Newton's method, surface area of solids of revolution, hyperbolic functions, and TNB frames. Based on our own teaching experience, we also added a brief new introductory section to the chapter on Techniques of Integration. We felt it makes sense to introduce students to some general integration strategies before diving into the standard techniques of integration by parts, partial fraction, and various substitutions.

Differential Equations

This book has a single robust section devoted to an overview of differential equations.

Pedagogical Features

Figures

Given the power of graphics software and the ease with which many students assimilate visual images, we devoted considerable time and deliberation to the figures in this book. Whenever possible, we let the figures communicate essential ideas using annotations reminiscent of an instructor's voice at the board. Readers will quickly find that the figures facilitate learning in new ways.

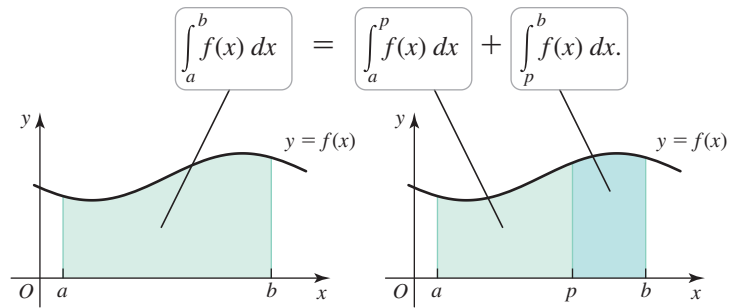


Figure 5.29

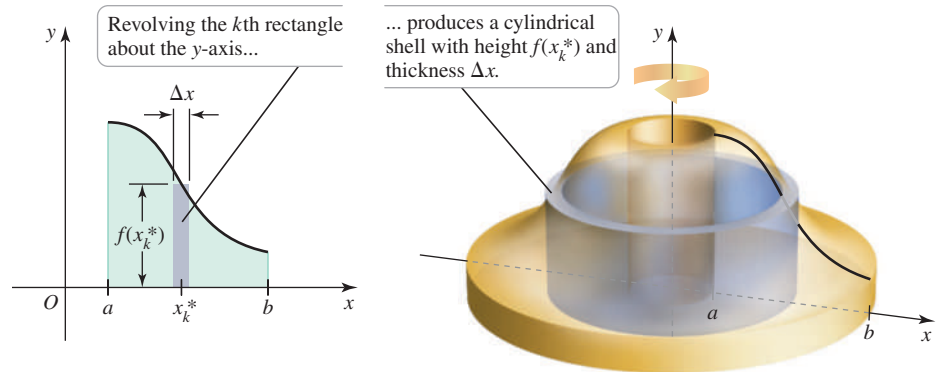


Figure 6.40

Quick Check and Margin Notes


The narrative is interspersed with *Quick Check* questions that encourage students to read with pencil in hand. These questions resemble the kinds of questions instructors pose in class. Answers to the *Quick Check* questions are found at the end of the section in which they occur. *Margin Notes* offer reminders, provide insight, and clarify technical points.

Guided Projects

The *Instructor's Resource Guide and Test Bank* contains 78 *Guided Projects*. These projects allow students to work in a directed, step-by-step fashion, with various objectives: to carry out extended calculations, to derive physical models, to explore related theoretical topics, or to investigate new applications of calculus. The *Guided Projects* vividly demonstrate the breadth of calculus and provide a wealth of mathematical excursions that go beyond the typical classroom experience. A list of suggested *Guided Projects* is included at the end of each chapter.

Technology

We believe that a calculus text should help students strengthen their analytical skills and demonstrate how technology can extend (not replace) those skills. Calculators and graphing utilities are additional tools in the kit, and students must learn when and when not to use them. Our goal is to accommodate the different policies about technology that various instructors may use.

Throughout the book, exercises marked with  indicate that the use of technology—ranging from plotting a function with a graphing calculator to carrying out a calculation using a computer algebra system—may be needed. See page 14 for information regarding our technology resource manuals covering Maple, Mathematica and Texas Instruments graphing calculators.

Content Highlights

In writing this text, we identified content in the calculus curriculum that consistently presents challenges to our students. We made organizational changes to the standard presentation of these topics or slowed the pace of the narrative to facilitate students' comprehension of material that is traditionally difficult. Two noteworthy modifications to the traditional table of contents for this course appear in the material for Calculus II and Calculus III.

Often appearing near the end of the term, the topics of sequences and series are the most challenging in Calculus II. By splitting this material into two chapters, we have given these topics a more deliberate pace and made them more accessible without adding significantly to the length of the narrative.

There is a clear and logical path through multivariate calculus, which is not apparent in many textbooks. We have carefully separated functions of several variables from vector-valued functions, so that these ideas are distinct in the minds of students. The book culminates when these two threads are joined in the last chapter, which is devoted to vector calculus.

Additional Resources

Instructor's Resource Guide and Test Bank

Bernard Gillett, University of Colorado at Boulder

This guide represents significant contributions by the textbook authors and contains a variety of classroom support materials for instructors.

- Seventy-eight *Guided Projects*, correlated to specific chapters of the text, can be assigned to students for individual or group work. The *Guided Projects* vividly demonstrate the breadth of calculus and provide a wealth of mathematical excursions that go beyond the typical classroom experience.
- *Lecture Support Notes* give an *Overview* of the material to be taught in each section of the text, and helpful classroom *Teaching Tips*. *Connections* among various sections of the text are also pointed out, and *Additional Activities* are provided.
- *Quick Quizzes* for each section in the text consist of multiple-choice questions that can be used as in-class quiz material or as Active Learning Questions. These Quick Quizzes can also be found at the end of each section in the interactive eBook.
- *Chapter Reviews* provide a list of key concepts from each chapter, followed by a set of chapter review questions.
- *Chapter Test Banks* consist of between 25 and 30 questions that can be used for in-class exams, take-home exams, or additional review material.
- *Learning Objectives Lists* and an *Index of Applications* are tools to help instructors gear the text to their course goals and students' interests.
- *Student Study Cards*, consisting of key concepts for both single-variable and multivariable calculus, are included for instructors to photocopy and distribute to their students as convenient study tools.
- *Answers* are provided for all exercises in the manual, including the *Guided Projects*.

Instructor's Solutions Manuals

Mark Woodard, Furman University
Single Variable Calculus (Chapters 1–10)
Multivariable Calculus (Chapters 8–14)

The *Instructor's Solutions Manual* contains complete solutions to all the exercises in the text.

Student's Solutions Manuals

Mark Woodard, Furman University
Single Variable Calculus (Chapters 1–10)
Multivariable Calculus (Chapters 8–14)

The *Student's Solutions Manual* is designed for the student and contains complete solutions to all the odd-numbered exercises in the text.

Just-in-Time Algebra and Trigonometry for Early Transcendentals Calculus, Fourth Edition

ISBN 0-321-67103-1 | 978-0-321-67103-5

Guntram Mueller and Ronald I. Brent, University of Massachusetts—Lowell

Sharp algebra and trigonometry skills are critical to mastering calculus, and *Just-in-Time Algebra and Trigonometry for Early Transcendentals Calculus* is designed to bolster these skills while students study calculus. As students make their way through calculus, this text is with them every step of the way, showing them the necessary algebra or trigonometry topics and pointing out potential problem spots. The easy-to-use table of contents has algebra and trigonometry topics arranged in the order in which students will need them as they study calculus.

Technology Resource Manual

TI-Graphing Calculator Manual by Elaine McDonald-Newman, Sonoma State University

This manual covers the TI-83 Plus/TI-84 Plus and TI-89, providing detailed guidance for integrating the graphing calculators throughout the course. The manual is available to instructors and students through www.pearsonglobaleditions.com/briggs.

TestGen[®]

TestGen[®] (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and testbank are available for download from www.pearsonglobaleditions.com/briggs.

PowerPoint[®] Lecture Slides

These PowerPoint slides contain key concepts, definitions, figures, and tables from the textbook. These files are available to qualified instructors through www.pearsonglobaleditions.com/briggs.

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1

Functions

Chapter Preview Mathematics is a language with an alphabet, a vocabulary, and many rules. Before beginning your calculus journey, you should be familiar with the elements of this language. Among these elements are algebra skills; the notation and terminology for various sets of real numbers; and the descriptions of lines, circles, and other basic sets in the coordinate plane. A review of this material is found in Appendix A. This chapter begins with the fundamental concept of a function and then presents the entire cast of functions needed for calculus: polynomials, rational functions, algebraic functions, exponential and logarithmic functions, and the trigonometric functions, along with their inverses. Before you begin studying calculus, it is important that you master the ideas in this chapter.

- 1.1 Review of Functions
- 1.2 Representing Functions
- 1.3 Inverse, Exponential, and Logarithmic Functions
- 1.4 Trigonometric Functions and Their Inverses

1.1 Review of Functions

Everywhere around us we see relationships among quantities, or **variables**. For example, the consumer price index changes in time and the temperature of the ocean varies with latitude. These relationships can often be expressed by mathematical objects called *functions*. Calculus is the study of functions, and because we use functions to describe the world around us, calculus is a universal language for human inquiry.

DEFINITION Function

A **function** f is a rule that assigns to each value x in a set D a *unique* value denoted $f(x)$. The set D is the **domain** of the function. The **range** is the set of all values of $f(x)$ produced as x varies over the entire domain (Figure 1.1).

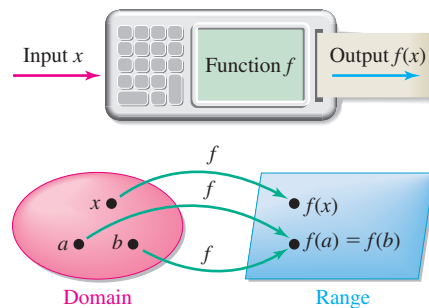


Figure 1.1

The **independent variable** is the variable associated with the domain; the **dependent variable** belongs to the range. The **graph** of a function f is the set of all points (x, y) in the xy -plane that satisfy the equation $y = f(x)$. The **argument** of a function is the expression on which the function works. For example, x is the argument when we write $f(x)$. Similarly, 2 is the argument in $f(2)$ and $x^2 + 4$ is the argument in $f(x^2 + 4)$.

QUICK CHECK 1 If $f(x) = x^2 - 2x$, find $f(-1)$, $f(x^2)$, $f(t)$, and $f(p - 1)$. ◀

The requirement that a function assigns a *unique* value of the dependent variable to each value in the domain is expressed in the vertical line test (Figure 1.2a). For example, the outside temperature as it varies over the course of a day is a function of time (Figure 1.2b).

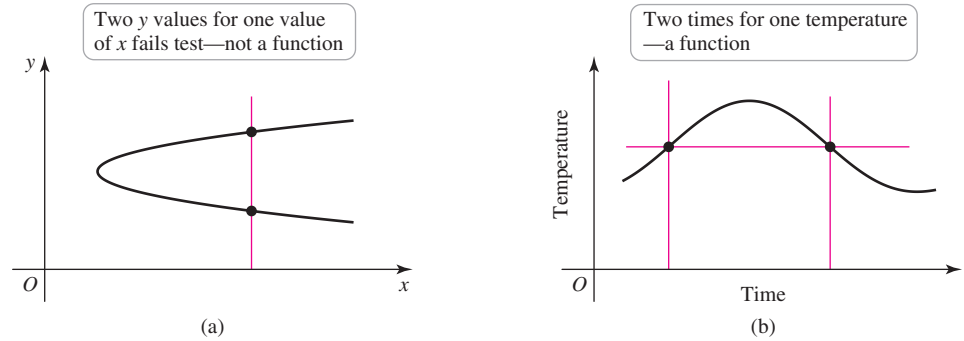


Figure 1.2

- ▶ If the domain is not specified, we take it to be the set of all values of x for which f is defined. We will see shortly that the domain and range of a function may be restricted by the context of the problem.
- ▶ A set of points or a graph that does *not* correspond to a function represents a **relation** between the variables. All functions are relations, but not all relations are functions.

Vertical Line Test

A graph represents a function if and only if it passes the **vertical line test**: Every vertical line intersects the graph at most once. A graph that fails this test does not represent a function.

EXAMPLE 1 Identifying functions State whether each graph in Figure 1.3 represents a function.

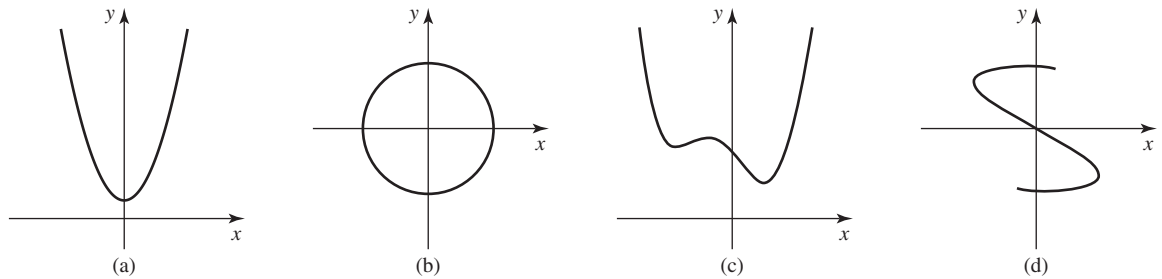


Figure 1.3

SOLUTION The vertical line test indicates that only graphs (a) and (c) represent functions. In graphs (b) and (d), there are vertical lines that intersect the graph more than once. Equivalently, there are values of x that correspond to more than one value of y . Therefore, graphs (b) and (d) do not pass the vertical line test and do not represent functions.

Related Exercises 11–12 ◀

- ▶ A window of $[a, b] \times [c, d]$ means $a \leq x \leq b$ and $c \leq y \leq d$.

EXAMPLE 2 Domain and range Graph each function with a graphing utility using the given window. Then state the domain and range of the function.

a. $y = f(x) = x^2 + 1$; $[-3, 3] \times [-1, 5]$

b. $z = g(t) = \sqrt{4 - t^2}$; $[-3, 3] \times [-1, 3]$

c. $w = h(u) = \frac{1}{u - 1}$; $[-3, 5] \times [-4, 4]$

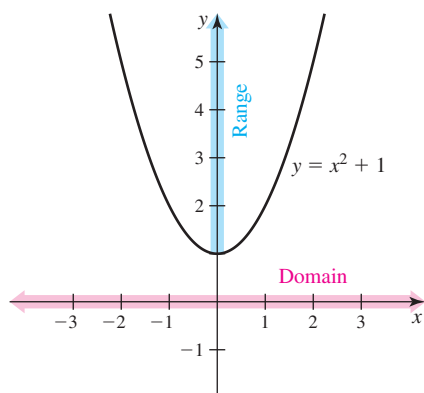


Figure 1.4

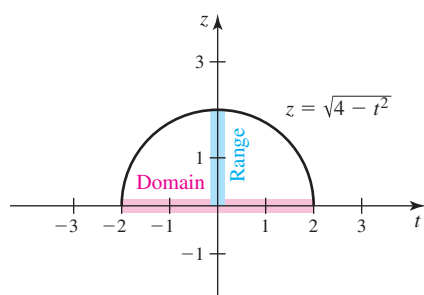


Figure 1.5

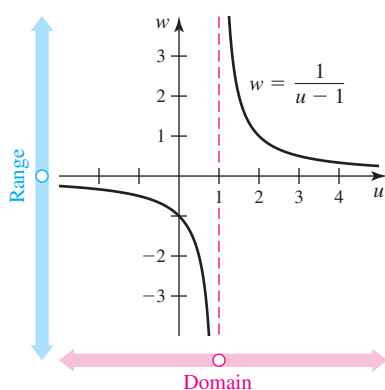


Figure 1.6

- The dashed vertical line $u = 1$ in Figure 1.6 indicates that the graph of $w = h(u)$ approaches a *vertical asymptote* as u approaches 1 and that w becomes large in magnitude for u near 1.

SOLUTION

- a. Figure 1.4 shows the graph of $f(x) = x^2 + 1$. Because f is defined for all values of x , its domain is the set of all real numbers, written $(-\infty, \infty)$ or \mathbb{R} . Because $x^2 \geq 0$ for all x , it follows that $x^2 + 1 \geq 1$ and the range of f is $[1, \infty)$.
- b. When n is even, functions involving n th roots are defined provided the quantity under the root is nonnegative (additional restrictions may also apply). In this case, the function g is defined provided $4 - t^2 \geq 0$, which means $t^2 \leq 4$, or $-2 \leq t \leq 2$. Therefore, the domain of g is $[-2, 2]$. By the definition of the square root, the range consists only of nonnegative numbers. When $t = 0$, z reaches its maximum value of $g(0) = \sqrt{4} = 2$, and when $t = \pm 2$, z attains its minimum value of $g(\pm 2) = 0$. Therefore, the range of g is $[0, 2]$ (Figure 1.5).
- c. The function h is undefined at $u = 1$, so its domain is $\{u: u \neq 1\}$, and the graph does not have a point corresponding to $u = 1$. We see that w takes on all values except 0; therefore, the range is $\{w: w \neq 0\}$. A graphing utility does *not* represent this function accurately if it shows the vertical line $u = 1$ as part of the graph (Figure 1.6).

Related Exercises 13–20 ◀

EXAMPLE 3 Domain and range in context At time $t = 0$, a stone is thrown vertically upward from the ground at a speed of 30 m/s. Its height above the ground in meters (neglecting air resistance) is approximated by the function $h = f(t) = 30t - 5t^2$, where t is measured in seconds. Find the domain and range of f in the context of this particular problem.

SOLUTION Although f is defined for all values of t , the only relevant times are between the time the stone is thrown ($t = 0$) and the time it strikes the ground, when $h = f(t) = 0$. Solving the equation $h = 30t - 5t^2 = 0$, we find that

$$\begin{aligned} 30t - 5t^2 &= 0 \\ 5t(6 - t) &= 0 && \text{Factor.} \\ 5t = 0 \quad \text{or} \quad 6 - t = 0 &&& \text{Set each factor equal to 0.} \\ t = 0 \quad \text{or} \quad t = 6. &&& \text{Solve.} \end{aligned}$$

Therefore, the stone leaves the ground at $t = 0$ and returns to the ground at $t = 6$. An appropriate domain that fits the context of this problem is $\{t: 0 \leq t \leq 6\}$. The range consists of all values of $h = 30t - 5t^2$ as t varies over $[0, 6]$. The largest value of h occurs when the stone reaches its highest point at $t = 3$ (halfway through its flight), which is $h = f(3) = 45$. Therefore, the range is $[0, 45]$. These observations are confirmed by the graph of the height function (Figure 1.7). Note that this graph is *not* the trajectory of the stone; the stone moves vertically.

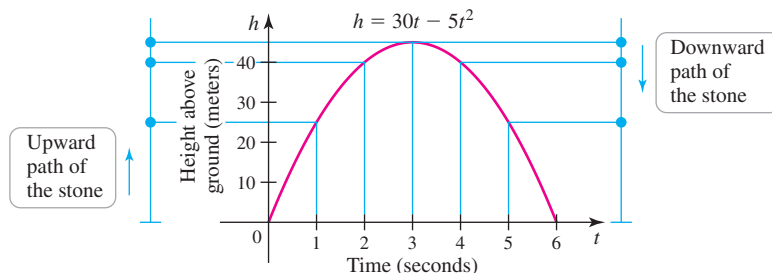


Figure 1.7

Related Exercises 21–24 ◀

QUICK CHECK 2 State the domain and range of $f(x) = (x^2 + 1)^{-1}$. ◀

Composite Functions

Functions may be combined using sums $(f + g)$, differences $(f - g)$, products (fg) , or quotients (f/g) . The process called *composition* also produces new functions.

- In the composition $y = f(g(x))$, f is the *outer function* and g is the *inner function*.

DEFINITION Composite Functions

Given two functions f and g , the composite function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$. It is evaluated in two steps: $y = f(u)$, where $u = g(x)$. The domain of $f \circ g$ consists of all x in the domain of g such that $u = g(x)$ is in the domain of f (Figure 1.8).

- You have now seen three different notations for intervals on the real number line, all of which will be used throughout the book:

- $[-2, 3)$ is an example of interval notation,
- $-2 \leq x < 3$ is inequality notation, and
- $\{x: -2 \leq x < 3\}$ is set notation.

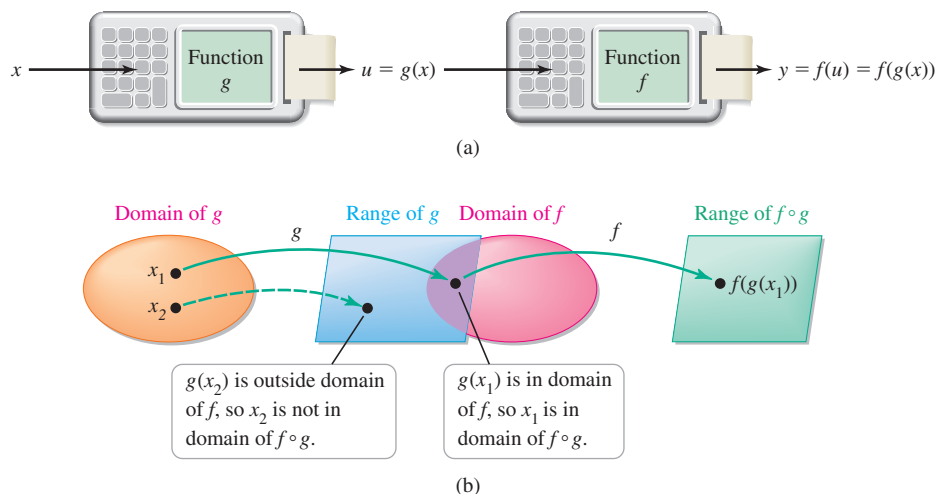


Figure 1.8

EXAMPLE 4 Composite functions and notation Let $f(x) = 3x^2 - x$ and $g(x) = 1/x$. Simplify the following expressions.

- a. $f(5p + 1)$ b. $g(1/x)$ c. $f(g(x))$ d. $g(f(x))$

SOLUTION In each case, the functions work on their arguments.

- a. The argument of f is $5p + 1$, so

$$f(5p + 1) = 3(5p + 1)^2 - (5p + 1) = 75p^2 + 25p + 2.$$

- b. Because g requires taking the reciprocal of the argument, we take the reciprocal of $1/x$ and find that $g(1/x) = 1/(1/x) = x$.

- c. The argument of f is $g(x)$, so

$$f(g(x)) = f\left(\frac{1}{x}\right) = 3\left(\frac{1}{x}\right)^2 - \left(\frac{1}{x}\right) = \frac{3}{x^2} - \frac{1}{x} = \frac{3 - x}{x^2}.$$

- d. The argument of g is $f(x)$, so

$$g(f(x)) = g(3x^2 - x) = \frac{1}{3x^2 - x}.$$

Related Exercises 25–36 ◀

EXAMPLE 5 Working with composite functions Identify possible choices for the inner and outer functions in the following composite functions. Give the domain of the composite function.

- a. $h(x) = \sqrt{9x - x^2}$ b. $h(x) = \frac{2}{(x^2 - 1)^3}$

SOLUTION

- a. An obvious outer function is $f(x) = \sqrt{x}$, which works on the inner function $g(x) = 9x - x^2$. Therefore, h can be expressed as $h = f \circ g$ or $h(x) = f(g(x))$. The domain of $f \circ g$ consists of all values of x such that $9x - x^2 \geq 0$. Solving this inequality gives $\{x: 0 \leq x \leq 9\}$ as the domain of $f \circ g$.

- Techniques for solving inequalities are discussed in Appendix A.

- b. A good choice for an outer function is $f(x) = 2/x^3 = 2x^{-3}$, which works on the inner function $g(x) = x^2 - 1$. Therefore, h can be expressed as $h = f \circ g$ or $h(x) = f(g(x))$. The domain of $f \circ g$ consists of all values of $g(x)$ such that $g(x) \neq 0$, which is $\{x: x \neq \pm 1\}$.

Related Exercises 37–40 ◀

EXAMPLE 6 More composite functions Given $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - x - 6$, find (a) $g \circ f$ and (b) $f \circ g$, and their domains.

SOLUTION

- a. We have

$$(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x}) = \underbrace{(\sqrt[3]{x})^2}_{f(x)} - \underbrace{\sqrt[3]{x}}_{f(x)} - 6 = x^{2/3} - x^{1/3} - 6.$$

Because the domains of f and g are $(-\infty, \infty)$, the domain of $f \circ g$ is also $(-\infty, \infty)$.

- b. In this case, we have the composition of two polynomials:

$$\begin{aligned} (g \circ g)(x) &= g(g(x)) \\ &= g(x^2 - x - 6) \\ &= \underbrace{(x^2 - x - 6)^2}_{g(x)} - \underbrace{(x^2 - x - 6)}_{g(x)} - 6 \\ &= x^4 - 2x^3 - 12x^2 + 13x + 36. \end{aligned}$$

The domain of the composition of two polynomials is $(-\infty, \infty)$.

Related Exercises 41–54 ◀

QUICK CHECK 3 If $f(x) = x^2 + 1$ and $g(x) = x^2$, find $f \circ g$ and $g \circ f$. ◀

EXAMPLE 7 Using graphs to evaluate composite functions Use the graphs of f and g in Figure 1.9 to find the following values.

- a. $f(g(3))$ b. $g(f(3))$ c. $f(f(4))$ d. $f(g(f(8)))$

SOLUTION

- a. The graphs indicate that $g(3) = 4$ and $f(4) = 8$, so $f(g(3)) = f(4) = 8$.

- b. We see that $g(f(3)) = g(5) = 1$. Observe that $f(g(3)) \neq g(f(3))$.

- c. In this case, $f(\underbrace{f(4)}_8) = f(8) = 6$.

- d. Starting on the inside,

$$f(\underbrace{g(f(8))}_6) = f(\underbrace{g(6)}_1) = f(1) = 6.$$

Related Exercises 55–56 ◀

EXAMPLE 8 Using a table to evaluate composite functions Use the function values in the table to evaluate the following composite functions.

- a. $(f \circ g)(0)$ b. $g(f(-1))$ c. $f(g(g(-1)))$

x	-2	-1	0	1	2
$f(x)$	0	1	3	4	2
$g(x)$	-1	0	-2	-3	-4

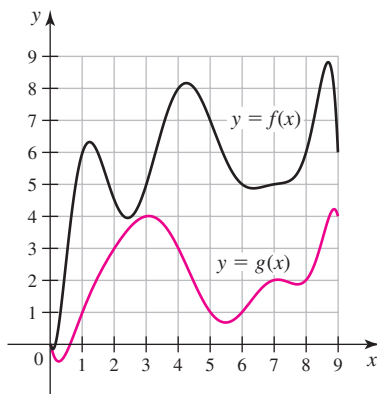


Figure 1.9